

New Exact Solutions for a Chiral Cosmological Model in 5D EGB Gravity

Maharaj S.D.,

*Astrophysics and Cosmology Research Unit
School of Mathematics, Statistics and Computer Science,
University of KwaZulu-Natal, Private Bag X54 001
Durban 4000, South Africa
Email: maharaj@ukzn.ac.za*

Beesham A.,

*Department of Mathematical Sciences, University of Zululand
Private Bag X1001, Kwa-Dlangezwa 3886, South Africa
Email: beeshama@unizulu.ac.za*

Chervon S.V.,

*Astrophysics and Cosmology Research Unit
School of Mathematics, Statistics and Computer Science,
University of KwaZulu-Natal, Private Bag X54 001
Durban 4000, South Africa and
Laboratory of Gravitation, Cosmology, Astrophysics,
Ulyanovsk State Pedagogical University,
100-years V.I. Lenin's Birthday square, Ulyanovsk 432071, Russia Email:
chervon.sergey@gmail.com*

Kubasov A.S.

*Laboratory of Gravitation, Cosmology, Astrophysics,
Ulyanovsk State Pedagogical University,
100-years V.I. Lenin's Birthday square, Ulyanovsk 432071, Russia Email:
as-kubasov@rambler.ru*

We consider a chiral cosmological model in the framework of Einstein-Gauss-Bonnet cosmology. Using a decomposition of the latter equations in such a way that the first chiral field is responsible for the Einstein part of the model, while the second field together with the kinetic interaction is connected with the Gauss-Bonnet part of the theory, we find new exact solutions for the 2-component chiral cosmological model with and without the kinetic interaction between fields.

Мы рассматриваем Киральную Космологическую Модель (ККМ) в рамках космологии ЭГБ. Используя разбиение уравнений ЭГБ таким образом, что первое киральное поле ответственно за эйнштейновскую часть модели, в то время как второе поле, вместе с кинетическим взаимодействием, связано с вкладом теории Гаусса-Бонне. Мы нашли новые точные решения для 2-х компонентной ККМ с кинетическим взаимодействием между полями и без него.

1 Introduction

The form of the potential of self-interaction for scalar field theory in cosmology is an interesting topic. The difference between scalar potentials in particle physics and those in cosmology has been stressed in the work [1]. Halliwell wrote "... we do not really know which theory of particle physics best describes the very early universe. One should therefore keep an open mind as to the form of $V(\phi)$."

To find exact solutions, usually one suggests that we know from high energy physics the scalar potential in the very early universe, and our task is to find the scale factor and the scalar field as functions of time. The work by Ellis and Madsen [2] was the first one to consider "the inverse problem" within the framework of cosmology. These authors [2] suggested to start from a given scale factor instead! Indeed, it is clear that the scale factor may be found from observational data. Then we may take into account this fact to find the potential and scalar field from the cosmological equations. This work was done and examples of exact solutions have been presented for pure scalar fields (without taking into account radiation which was also considered there). Further, this approach was developed in the works [3, 4]. In our study, we will use such an approach, the so-called "fine tuning of the potential method", to find new solutions in cosmology based on Einstein-Gauss-Bonnet (EGB) gravity in five-dimensional (5D) spacetime.

The feature of our approach is in the fact that we use the chiral cosmological model (CCM) [5] reduced to a two dimensional target space. That is, for our consideration in the present contribution, we consider two scalar fields with or without the kinetic interaction term. Such an approach has been considered, for example, with the aim of exact solutions construction in [6, 7] and for studying inflation in $f(R)$ gravity in [8]. The set of exact solutions in the 2-component CCM for 5D EGB gravity for the Emergent Universe was found in the work [9]. The evolution of the scale factor was taken in the form $a(t) = A(\beta + e^{\alpha t})^m$. Also in [9], it was mentioned about the possibility to extend the method to exponential, power-law and other kinds of solutions.

In the present investigation, we use once again the method of the decomposition of the field-gravity equations and the definition of special ansatzes. The method was described for GR in [10] and for the case of EGB gravity in [9]. We study new possibilities for exact solution construction for various types of evolutions, including power-law, power-law-exponential and hyperbolic scale factors. We stress that the obtained solutions are 5D ones, and that there is no way to compare exact solutions in 5D EGB with those in Friedmann cosmology based on Einstein's theory. It is not possible to define cosmological parameters from 5D EGB cosmology without compactification of the fifth dimension. However, this is not the main aim of this paper, and will be a subject for future investigation.

2 Basic equations of the model

Chiral cosmological fields are considered as the source of gravitation in the EGB model. The action of the model is:

$$S = \int d^5x \sqrt{-g} \left(\frac{1}{2}R + \frac{1}{2}\bar{\alpha}R_{GB} + \frac{1}{2}h_{AB}(\varphi)\varphi_{,a}^A\varphi_{,b}^Bg^{ab} - V(\varphi) \right), \quad (1)$$

where R_{GB} is the Lovelock tensor defined as $R_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$ and $\bar{\alpha}$ is the Gauss-Bonnet parameter. Other notations correspond to those in [10].

We will study cosmology in 5D space-time with the Friedmann-Robertson-Walker (FRW)-type metric

$$dS^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - \epsilon r^2} + r^2(d\theta^2 + \sin^2\theta(d\varphi^2 + \sin^2\varphi d\chi^2)) \right), \quad (2)$$

and we will take a CCM with the 2-component diagonal metric of a target-space

$$ds_{ts} = h_{11}d\phi^2 + h_{22}(\phi, \psi)d\psi^2, \quad (3)$$

as the source of gravity.

The system of fields and Einstein's equations may be displayed in the following form [9]:

$$H^2 + \frac{\epsilon}{a^2} + \bar{\alpha} \left(H^2 + \frac{\epsilon}{a^2} \right)^2 = \frac{1}{6} \left(\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} h_{22}(\phi, \psi) \dot{\psi}^2 + V(\phi, \psi) \right), \quad (4)$$

$$\left[1 + 2\bar{\alpha} \left(H^2 + \frac{\epsilon}{a^2} \right) \right] \left(\dot{H} - \frac{\epsilon}{a^2} \right) = -\frac{1}{3} \left(h_{11} \dot{\phi}^2 + h_{22}(\phi, \psi) \dot{\psi}^2 \right), \quad (5)$$

$$h_{11} \ddot{\phi} + 4H h_{11} \dot{\phi} - \frac{1}{2} \frac{\partial h_{22}}{\partial \phi} \dot{\psi}^2 + \frac{\partial V}{\partial \phi} = 0, \quad h_{11} = \text{constant}, \quad (6)$$

$$h_{22}(\phi, \psi) \ddot{\psi} + \dot{h}_{22}(\phi, \psi) \dot{\psi} + 4H h_{22}(\phi, \psi) \dot{\psi} - \frac{1}{2} \frac{\partial h_{22}}{\partial \psi} \dot{\psi}^2 + \frac{\partial V}{\partial \psi} = 0. \quad (7)$$

Our main attention will be to find exact solutions and we consider the case of the FRW spatially-flat universe ($\epsilon = 0$). The kinetic energy and potential can be easily obtained from the basic equations (4)-(7) and read:

$$-\frac{2}{3}K(t) = [1 + 2\bar{\alpha}H^2] \dot{H} = -\frac{1}{3} \left(h_{11} \dot{\phi}^2 + h_{22}(\phi, \psi) \dot{\psi}^2 \right), \quad (8)$$

$$\frac{V}{6} = H^2 + \frac{1}{4} \dot{H} + \bar{\alpha} H^2 \left(H^2 + \frac{1}{2} \dot{H} \right). \quad (9)$$

We may solve the system (4)–(7) using the decomposition suggested in [9]

$$h_{11} \dot{\phi}^2 = -3\dot{H}, \quad h_{22}(\phi, \psi) \dot{\psi}^2 = -6\bar{\alpha}H^2 \dot{H}, \quad V(\phi, \psi) = V_1(\phi) + e^{f(\phi)} V_2(\psi), \quad (10)$$

where

$$V_1(\phi(t)) = 6H^2 + \frac{3}{2} \dot{H}, \quad e^{f(\phi(t))} V_2(\psi(t)) = 6\bar{\alpha}H^2 \left(H^2 + \frac{1}{2} \dot{H} \right). \quad (11)$$

Here $f(\phi)$ is a function on time, defined from compatibility later on, and we set h_{11} equal to unity: $h_{11} = 1$. The relation between the metric coefficient h_{22} and $f(\phi)$ is dependent on the law of evolution of the Hubble parameter and is determined from the equation:

$$\frac{\partial \ln(h_{22})}{\partial t} = -\frac{2H^2 + \dot{H}}{\dot{H}} \frac{\partial f}{\partial t}. \quad (12)$$

For further investigation we will use the equation of state in the following form

$$\omega = -1 - \frac{\dot{H}}{2H^2} \left(\frac{1 + 2\bar{\alpha}H^2}{1 + \bar{\alpha}H^2} \right). \quad (13)$$

Thus we have one free parameter, namely, a kinetic interaction h_{22} as a function of the fields. It is worthwhile to mention that it is difficult to obtain the kinetic interaction term from observations [11], but numerically the reconstruction of the target space metric component h_{22} and the potential of interaction $V(\phi, \psi)$ was carried out in the work [12].

3 Exact solutions without kinetic interaction between fields

In the work [9] it was proved that, using the decomposition (10)-(11), exact solutions for the 2-component CCM for the emergent universe can be obtained only for chiral fields without kinetic interaction between them. In our present consideration, we will show that both cases are possible: without kinetic interaction and with it. In the next section, we will present the solutions with kinetic interaction between the chiral fields. Here we will start with the case when the kinetic interaction between fields is absent. For this purpose we set $h_{11} = h_{22} = 1$. After that simplification we can represent the "ansatz solution" as:

$$\dot{\phi}^2 = -3\dot{H}, \quad (14)$$

$$\dot{\psi}^2 = -6\bar{\alpha}H^2\dot{H}, \quad (15)$$

$$V_1(\phi) = 6H^2 + \frac{3}{2}\dot{H}, \quad (16)$$

$$V_2(\psi) = 6\bar{\alpha}H^2 \left(H^2 + \frac{1}{2}\dot{H} \right). \quad (17)$$

Substitution of equations (14)-(17) into the field equations (6)-(7) gives us identities. Note that for $h_{22} = 1$, we obtain $f(\phi) = \text{const.}$, and the term $e^{f(\phi)}$ may be included in $V_2(\psi)$.

Thus to construct exact solutions, we may use the evolutionary law of the universe. That is, for a given scale factor $a(t)$ (or Hubble parameter $H(t)$), we can obtain the potential and the kinetic energy as functions of time. Chiral fields may be defined from equations (14)-(15) in quadratures, or in elementary functions depending on the given scale factor.

3.1 Power-law evolution

Power-law evolution is very important for a variety of reasons, e.g., such exact solutions can solve the horizon, flatness and perturbation-spectrum problems in FRW cosmology. Thus we choose the scale factor as

$$a(t) = At^m, \quad A = \text{const.}, \quad A > 0, \quad m > 1. \quad (18)$$

Under these conditions above we have $H_{pl} = m/t$ and the acceleration of the Universe is always positive : $\frac{\ddot{a}}{a} = H^2 + \dot{H} = \frac{m(m-1)}{t^2} > 0$.

The solution for ϕ from (14) is

$$\phi - \phi_* = \pm\sqrt{3m} \ln t. \quad (19)$$

Hereinafter the letters with a subscript-star mean constants of integration. The second field ψ can be defined by integrating (15) and it gives

$$\psi - \psi_* = \mp \frac{\sqrt{6\bar{\alpha}m^3}}{t}. \quad (20)$$

Using the obtained solutions for the chiral fields ϕ and ψ , we can reconstruct the potentials V_1 and V_2 from their dependence on time t . Finally we obtain

$$V_1(\phi) = \frac{3m}{2}(4m-1) \exp \left(\mp \frac{2(\phi - \phi_*)}{\sqrt{3m}} \right), \quad (21)$$

$$V_2(\psi) = \frac{2m-1}{12\bar{\alpha}m^3} (\psi - \psi_*)^4. \quad (22)$$

The equation of state (13) for the solution (19)-(22) is

$$\omega = -1 + m^{-1} \left(1 + \frac{\bar{\alpha}m^2}{t^2 + \bar{\alpha}m^2} \right). \quad (23)$$

3.2 Power-law-exponential evolution

The scale factor for power-law-exponential evolution of the universe is

$$a(t) = At^m e^{\lambda t}, \quad \lambda = \text{const.}, \quad \lambda > 0, \quad m > 1. \quad (24)$$

We come across such type of evolution of the universe in brane cosmology [13]. The solution for ϕ from (14) is

$$\phi - \phi_* = \pm \sqrt{3m} \ln t. \quad (25)$$

It is interesting to mention that we have got the same solution for ϕ as in the case of power-law evolution. The reason is that the Hubble parameters H defer by the constant:

$$H_{pl} = \frac{m}{t}; \quad H_{pl-exp} = \frac{m}{t} + \lambda.$$

Therefore we have the same right hand side in (14) for finding the field ϕ . We note, that the acceleration is positive: $\frac{\ddot{a}}{a} = \frac{m(m-1)}{t^2} + \frac{2m\lambda}{t} + \lambda^2 > 0$.

Evidently, from (15), we obtain another result because H_{pl-exp} is involved in the equation. Thus, performing integration (15), we find

$$\psi - \psi_* = \pm \sqrt{6\bar{\alpha}m} \left(-\frac{m}{t} + \lambda \ln t \right). \quad (26)$$

Using the obtained solution for the chiral field ϕ , we can make the reconstruction of the potential V_1 from its dependence on time t . Thus we obtain

$$V_1(\phi)/3 = \left(2m - \frac{1}{2} \right) m \exp \left(\mp 2 \frac{\phi - \phi_*}{\sqrt{3m}} \right) + 4m\lambda \exp \left(\mp \frac{\phi - \phi_*}{\sqrt{3m}} \right) + 2\lambda^2. \quad (27)$$

For the second field ψ it is impossible to make a reconstruction to get the explicit dependence V_2 on ψ . Therefore we can present the expression for V_2 in terms of t

$$V_2 = 6\bar{\alpha} \left(\frac{m}{t} + \lambda \right)^2 \left(\frac{m(m-1/2)}{t^2} + 2\frac{m\lambda}{t} + \lambda^2 \right). \quad (28)$$

With the help of equation (26) we can express m/t in terms of ψ and $\ln t$

$$m/t = F(\psi, t) = \lambda \ln t \mp \frac{\psi}{\sqrt{6\bar{\alpha}m}}.$$

Then after substituting $F(\psi, t)$ into (28), one can obtain

$$V_2(\psi, t) = 6\bar{\alpha} (F(\psi, t) + \lambda)^2 \left(\left(1 - \frac{1}{2m} \right) F(\psi, t)^2 + 2\lambda F(\psi, t) + \lambda^2 \right). \quad (29)$$

3.3 Sin-Hyperbolic evolution

As an example of hyperbolic evolution of the Universe let us consider the scale factor

$$a(t) = A \sinh \lambda t, \quad \lambda = \text{const.} > 0. \quad (30)$$

Such type of evolution was studied in [2] The solution for ϕ from (14) can be written as

$$\phi - \phi_* = \pm \frac{1}{2} \sqrt{3} \ln \left(\frac{\cosh \lambda t - 1}{\cosh \lambda t + 1} \right). \quad (31)$$

what is equivalent to

$$\phi - \phi_* = \pm \sqrt{3} \ln \left(\tanh \left(\frac{\lambda t}{2} \right) \right). \quad (32)$$

The second field ψ can be defined by integrating (15) and it gives

$$\psi - \psi_* = \mp \frac{\lambda \sqrt{6\bar{\alpha}}}{\sinh \lambda t}. \quad (33)$$

Using the obtained solutions for the chiral fields ϕ and ψ , we can reconstruct the potentials V_1 and V_2 from their dependence on time t . Finally we obtain

$$V_1(\phi) = \lambda^2 \left(6 + 4.5 \sinh^2 \frac{(\phi - \phi_*)}{\sqrt{3}} \right), \quad (34)$$

$$V_2(\psi) = 6\bar{\alpha}\lambda^4 \left(1 + \frac{(\psi - \psi_*)^2}{6\bar{\alpha}\lambda^2} \right) \left(1 + \frac{1}{2} \frac{(\psi - \psi_*)^2}{6\bar{\alpha}\lambda^2} \right). \quad (35)$$

Considering the very early stages of the evolution of the universe, we let time t to zero: $t \rightarrow 0$. Then, it is easy to see that both fields tend to infinity: $\phi, \psi \rightarrow \mp \infty$. The same will be true for the potential: $V = V_1 + V_2$, $V \rightarrow \infty$.

3.4 Cos-Hyperbolic evolution

We consider nonsingular evolution of the universe with the scale factor

$$a(t) = A \cosh \lambda t, \quad \lambda = \text{const.} > 0. \quad (36)$$

We note, that we have once again accelerated expansion with $\ddot{a} = A\lambda^2 \cosh \lambda t > 0$.

In the work [2], such type of evolution was considered, but it was necessary to add the dust part to avoid imaginary values of a scalar field. In our case, for the evolution (36), $\dot{H} > 0$, and we can see from (14)-(15) that if we want to work with real chiral fields, we have to choose the negative sign for h_{11} and h_{22} , i.e., both fields should be phantom ones. Let $h_{11} = -1$ and $h_{22} = -1$. Taking into account these relations, one can find the solution for ϕ from (14)

$$\phi - \phi_* = \pm 2\sqrt{3} \arctan \left(e^{\lambda t} \right). \quad (37)$$

The second field ψ can be defined by integrating (15), and it gives

$$\psi - \psi_* = \mp \frac{\lambda \sqrt{6\bar{\alpha}}}{\cosh \lambda t}. \quad (38)$$

Using the obtained solutions for the chiral fields ϕ and ψ , we can reconstruct the potentials V_1 and V_2 from their dependence on time t . Finally we obtain

$$V_1(\phi) = \frac{3}{2}\lambda^2 \left(3 \cos^2 \frac{(\phi - \phi_*)}{\sqrt{3}} + 1 \right), \quad (39)$$

$$V_2(\psi) = 6\bar{\alpha}\lambda^4 \left(1 - \frac{(\psi - \psi_*)^2}{6\bar{\alpha}\lambda^2} \right) \left(1 - \frac{1}{2} \frac{(\psi - \psi_*)^2}{6\bar{\alpha}\lambda^2} \right). \quad (40)$$

In this case the fields and the potential will tend to constants in the very early stages of the evolution of the universe when $t \rightarrow 0$.

It is worthwhile to mention that the analog of such a solution for the model with kinetic interaction contradicts the suggested decomposition or may contain imaginary fields.

4 Exact solutions with kinetic interaction between fields

In this section we give examples when exact solutions exist with kinetic interaction between scalar fields. We recall that it was impossible to get exact solutions in the emergent universe in EGB gravity as suggested in [9] and under that decomposition.

4.1 Power-law expansion

For the power-law expansion (18), by suggesting

$$h_{22} = Ct^{2m-1}, \quad C = \text{const.} \quad (41)$$

we obtain the solution for the chiral fields

$$\phi - \phi_* = \sqrt{3m} \ln t, \quad \psi - \psi_* = \sqrt{6\bar{\alpha} \frac{m^3}{C}} \left(-\frac{2}{2m+1} \right) t^{-m-\frac{1}{2}}, \quad m \neq -1/2. \quad (42)$$

The reconstruction of potentials gives

$$V_1(\phi) = m(6m - 3/2) \exp \left(-2 \frac{\phi - \phi_*}{\sqrt{3m}} \right), \quad (43)$$

$$V_2(\psi) = 3\bar{\alpha}m^3(2m-1) \left(\frac{(2m+1)\sqrt{C}}{2\sqrt{6\bar{\alpha}m^3}} (\psi - \psi_*) \right)^{\frac{5}{m+\frac{1}{2}}}. \quad (44)$$

Thus for the power-law kinetic interaction (41), we found the chiral fields to be dynamic and both parts of the potential.

4.2 Power-law-exponential expansion

The scale factor for power-law-exponential inflation has the form:

$$a(t) = At^m e^{\lambda t}. \quad (45)$$

Kinetic and potential energy can be written in this case as follows:

$$K(t) = \frac{3}{2} \frac{m}{t^2} + 3\bar{\alpha} \frac{m}{t^2} \left(\frac{m^2}{t^2} + 2\lambda \frac{m}{t} + \lambda^2 \right), \quad (46)$$

$$V(t) = 6 \left(\frac{m}{t} + \lambda \right)^2 - \frac{3}{2} \frac{m}{t^2} + 6\bar{\alpha} \left(\frac{m}{t} + \lambda \right)^2 \left(\left(\frac{m}{t} + \lambda \right)^2 - \frac{1}{2} \frac{m}{t^2} \right). \quad (47)$$

In the model under consideration, we can obtain the exact solution for power-law-exponential evolution using the ansatz (10) and the relation (12). We choose the term responsible for kinetic interaction of the fields to have the form

$$h_{22}(\phi) = 2 \frac{\lambda}{m^2} \left(m \exp \left(-\frac{\phi - \phi_*}{\pm \sqrt{3m}} \right) + \lambda \right)^2. \quad (48)$$

The solution for chiral fields dynamic described by

$$\phi - \phi_* = \pm \sqrt{3m} \ln t, \quad \psi - \psi_* = \pm \sqrt{3\bar{\alpha}m} \frac{m}{\sqrt{\lambda}} \ln t. \quad (49)$$

Below we will use the following notation

$$\tilde{\phi} = \frac{\phi - \phi_*}{\pm \sqrt{3m}}, \quad \tilde{\psi} = \pm \sqrt{\frac{\lambda}{3\bar{\alpha}m}} \frac{\psi - \psi_*}{m}.$$

The potential's components are

$$V_1(\phi) = 6 \left(m e^{-\tilde{\phi}} + \lambda \right)^2 - \frac{3}{2} e^{-\tilde{\phi}}, \quad (50)$$

$$V_2(\psi) = 6\bar{\alpha} \left(me^{-\tilde{\psi}} + \lambda \right)^2 \left(\left(me^{-\tilde{\psi}} + \lambda \right)^2 - \frac{1}{2} me^{-\tilde{\psi}} \right) e^{-f(\psi)}, \quad (51)$$

$$e^{f(\psi)} = \left(\frac{e^{-\tilde{\psi}} + \frac{\lambda}{m}}{\left(3e^{-2\tilde{\psi}} + 4e^{-\tilde{\psi}} \frac{\lambda}{m} + \left(\frac{\lambda}{m} \right)^2 \right)^{1/3}} \right)^{2/m}, \exp \left[-\frac{2\sqrt{2}}{3m} \arctan \left(\frac{m}{\lambda 2\sqrt{2}} \left(6e^{-\tilde{\psi}} + 4\frac{\lambda}{m} \right) \right) \right], \quad (52)$$

$$e^{f(\phi)} = \left(\frac{e^{-\tilde{\phi}} + \frac{\lambda}{m}}{\left(3e^{-2\tilde{\phi}} + 4e^{-\tilde{\phi}} \frac{\lambda}{m} + 2 \left(\frac{\lambda}{m} \right)^2 \right)^{1/3}} \right)^{2/m}, \exp \left[-\frac{2\sqrt{2}}{3m} \arctan \left(\frac{m}{\lambda 2\sqrt{2}} \left(6e^{-\tilde{\phi}} + 4\frac{\lambda}{m} \right) \right) \right], \quad (53)$$

$$V(\phi, \psi) = V_1(\phi) + e^{f(\phi)} V_2(\psi). \quad (54)$$

In the formulae above $f(\phi)$ and $f(\psi)$ have the same representation in terms of time t . Let us note that, to obtain $V(\phi, \psi)$ in the correct form (54), we must express the three functions V_1 , e^{ϕ} and V_2 as functions of their arguments, ϕ , ϕ and ψ , respectively. Therefore, for example, to express the function V_2 as a function on t we will use the dependence V_2 on t (from (47)) and then, defining the cosmic time t as the function on ψ from the second formula in (49), we insert the result in $V_2(t)$ to obtain the desired dependence $V_2(\psi)$.

4.3 Sin-Hyperbolical expansion

We choose the scale factor as

$$a(t) = A \sinh(\lambda t). \quad (55)$$

The dynamic chiral fields are described by

$$\phi - \phi_* = \pm 2\sqrt{3} \tanh^{-1} \left(e^{\lambda t} \right), \quad (56)$$

$$\psi - \psi_* = \pm \sqrt{6\bar{\alpha}} \lambda \ln(\sinh(\lambda t)). \quad (57)$$

We introduce additional functions

$$\Theta(\phi) = \ln \left(\tanh \left(\exp \left(\pm \frac{\phi - \phi_*}{2\sqrt{3}} \right) \right) \right) \quad (58)$$

and

$$\chi(\psi) = \sinh^{-1} \left(\exp \left(\pm \frac{\psi - \psi_*}{\lambda \sqrt{6\bar{\alpha}}} \right) \right). \quad (59)$$

Then the solution takes the form

$$V_1(\phi) = \frac{9\lambda^2}{2} \sinh^{-2} \Theta(\phi) + 6\lambda^2, \quad (60)$$

$$f(\phi) = \left(\frac{3}{\sinh^2(\Theta(\phi))} + 2 \right)^{1/3}, \quad (61)$$

$$V_2(\psi) = 3\bar{\alpha}\lambda^4 \frac{\cosh^2(\chi(\psi))(\cosh^2(\chi(\psi)) + 1)}{(2\cosh^2(\chi(\psi)) + 1)^{1/3}} \sinh^{-5/3}(\chi(\psi)). \quad (62)$$

Thus we proved that the singular solution of sinh-type may be presented by the model with kinetic interaction of fields.

5 Discussions

We considered power-law, power-law-exponential and hyperbolic types for the scale factor in EGB cosmology with a two-field CCM. First of all, we found the exact solutions for scalar fields without kinetic interaction in analogy with the emergent universe scenario [9]. The decomposition used in that work seemed to indicate that solutions with kinetic interaction are impossible. Nevertheless, when we extended our approach for scale factor evolution different from that of the emergent universe scenario, solutions with kinetic interaction were found.

If we can find a reasonable way of compactification of the 5th dimension, and make sure that spectral parameters can be calculated without perturbation analysis, then the exact solutions we found can be confronted with observational data. We were not looking for any relation with $f(R)$ theories, but note that a connection with R^2 theory is important if one wishes to study inflation, but these are topics for future investigation.

Perhaps dark energy could be described by some of our solutions obtained.

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